Mathematics Entrance Examination

Practice set (A)

Formula List

For the equation $ax^2 + bx + c = 0$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Curved surface area, A, of cylinder of radius r, height h. $A = 2\pi rh$

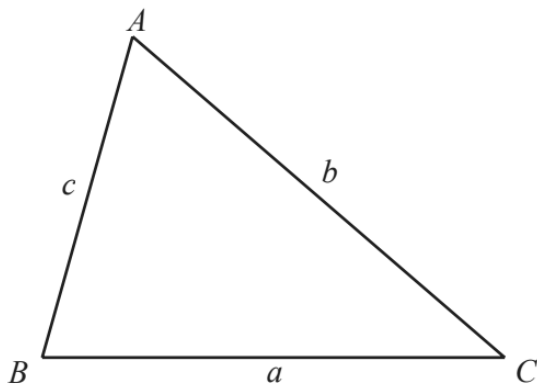Curved surface area, A, of cone of radius r, sloping edge l. $A = \pi rl$

Curved surface area, A, of sphere of radius r. $A = 4\pi r^2$

Volume, V, of pyramid, base area A, height h. $V = \dfrac{1}{3}Ah$

Volume, V, of cylinder of radius r, height h. $V = \pi r^2 h$

Volume, V, of cone of radius r, height h. $V = \dfrac{1}{3}\pi r^2 h$

Volume, V, of sphere of radius r. $V = \dfrac{4}{3}\pi r^3$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$

$\text{Area} = \dfrac{1}{2}bc \sin A$

Questions

Q1.

(a) Find the value of 12 ÷ (2 − 5)

....
(1)

(b) Find the value of 6^4

....
(1)

(c) Write down the prime number whose value is nearest to 33

....
(1)

(d) Find the cube root of 343

....
(1)

(Total for question = 4 marks)

Q2.

The perimeter of a triangle is 90 cm.
The lengths of the sides of the triangle are in the ratios 3 : 5 : 7.
Work out the length of the longest side of the triangle.

.... cm

(Total for question = 5 marks)

Q3.

In January 2007 the population of Canada was 32 million.
7 million of these Canadian people spoke French as their first language.

(a) Express 7 million as a percentage of 32 million.
Give your answer correct to 1 decimal place.

.... %

(2)

Between January 2007 and January 2009 the population of Canada increased by 15%.

(b) Increase 32 million by 15%.
Give your answer correct to the nearest million.

.... million

(2)

(c) In one year the population of a small island increased by 20% to 624. What was the population before the increase?

....

(2)

(Total for question is 6 marks)

Q4. Simplify the following surd showing full working.

$$\sqrt{50} + 2\sqrt{32}$$

....
(2)

(b) Show that $(4 + \sqrt{12})(5 - \sqrt{3}) = 14 + 6\sqrt{3}$
Show each stage of your working clearly.

(3)

(Total for question = 5 marks)

Q5.

(a) Factorise $14x - 35$

....

(1)

(b) Expand and simplify $3(2c - 5) - 2(c - 4)$

....

(2)

(c) Simplify $(4e^3)^2$

....

(2)

(d) Expand and simplify $(a + 5)(2a - 1)$

....

(2)

(Total for question = 7 marks)

Q6.

Given that y is positive, make y the subject of $y = \sqrt{ay^2 + n}$

Show clear algebraic working.

$y =$ ....

(Total for Question is 5 marks)

Q7.

(a) Factorise $4x^2 - 1$

....

(2)

(b) Solve $\dfrac{4}{2x+1} + \dfrac{1}{4x^2 - 1} = 3$

Show clear algebraic working.

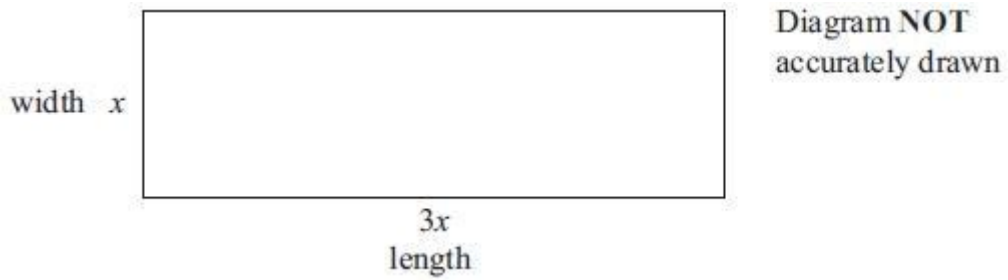..

(4)

(Total for question = 6 marks)

Q8.

The diagram shows a rectangular playground of width x metres and length $3x$ metres.

width x

$3x$
length

Diagram **NOT** accurately drawn

The playground is extended, by adding 10 metres to its width and 20 metres to its length, to form a larger rectangular playground.
The area of the larger rectangular playground is double the area of the original playground.

(a) Show that $3x^2 - 50x - 200 = 0$

(3)

(b) Calculate the area of the original playground.

....m^2

(5)

(Total for question = 8 marks)

Q9.

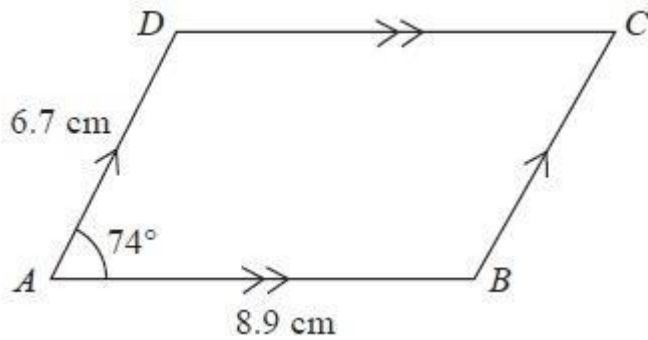

Diagram **NOT** accurately drawn

ABCD is a parallelogram.
AB = 8.9 cm.
AD = 6.7 cm.
Angle *BAD* = 74°

Calculate the area of parallelogram *ABCD*.
Give your answer correct to 3 significant figures.

.... cm^2

(Total for Question is 3 marks)

Q10.
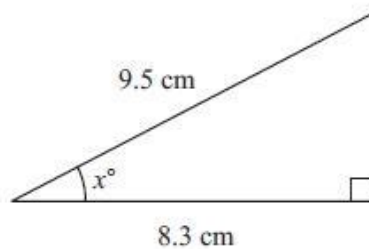
Diagram **NOT** accurately drawn

Work out the value of *x*.
Give your answer correct to 1 decimal place.

x = ...
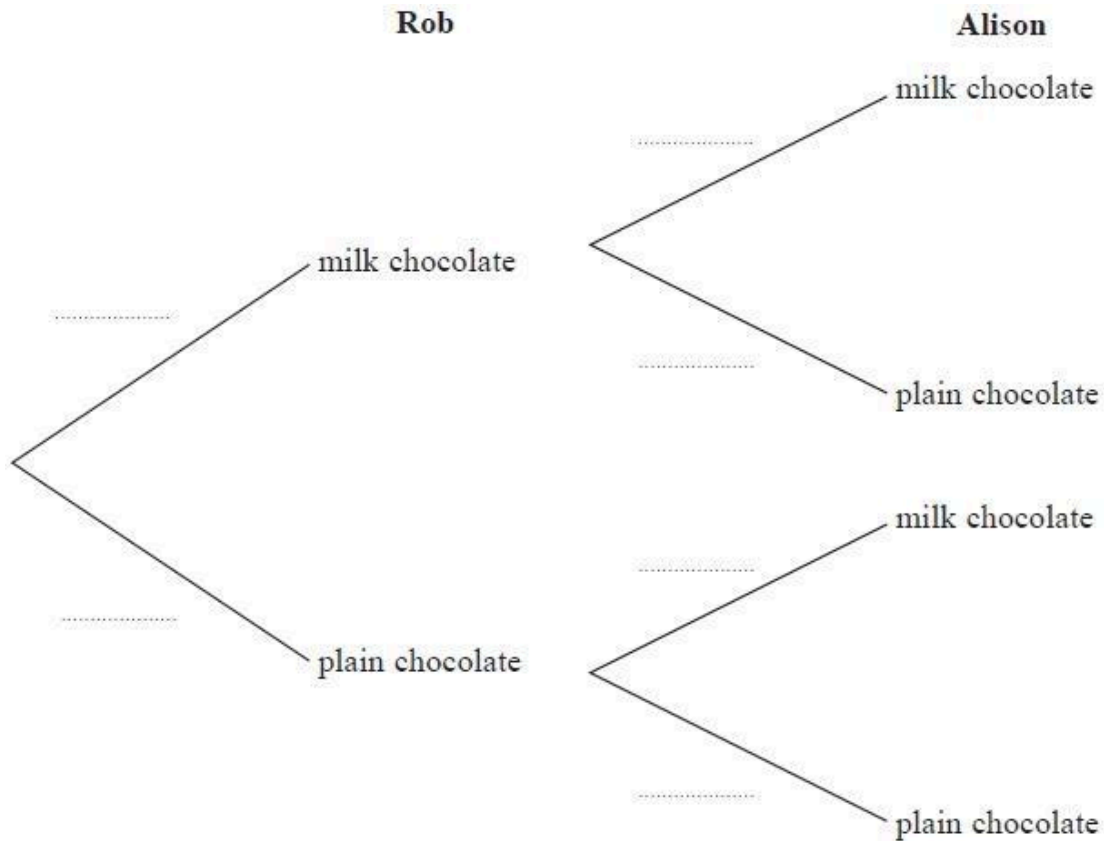
(Total for question = 3 marks)

Q11.

There are 6 milk chocolates and 4 plain chocolates in a box.
Rob takes at random a chocolate from the box and eats it.
Then Alison takes at random a chocolate from the box and eats it.

(a) Complete the probability tree diagram.

Rob　　　　　　　　　　　　**Alison**

milk chocolate

.... milk chocolate

.... plain chocolate

.... plain chocolate

.... milk chocolate

.... plain chocolate

(3)

(b) Work out the probability that there are now exactly 3 plain chocolates in the box.

....
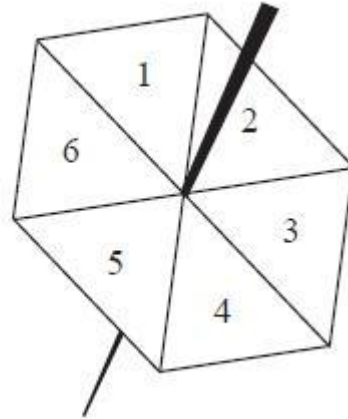
(3)

(Total for question = 6 marks)

Q12.

Becky has a biased 6-sided spinner.
She spins the spinner 25 times.
She records the score for each spin.
The table shows information about her scores.

Score	Frequency
1	9
2	6
3	3
4	2
5	1
6	4

(a) Find her median score.

....

(2)

(b) Work out her mean score.

....

(3)

(Total for question = 5 marks)

Q13.

Three positive whole numbers are all different.
The numbers have a median of 8 and a mean of 6
Find the three numbers.

....

(Total for question = 2 marks)

Mark Scheme (questions are out of order)

Q1.

Question	Working	Answer	Mark	Notes
(a)		− 4	1	B1
(b)		1296	1	B1
(c)		31	1	B1
(d)		7	1	B1
				Total 4 marks

Q2.

Question Number	Working	Answer	Mark	Notes	
	3+5+7 or 15		3	M1	15 may be denominator of fraction or coefficient in an equation such as $15x = 90$
	90 ÷ (3+5+7) or 90 ÷ "15" or 6 or $\frac{7}{15}$ oe			M1	dep
		42		A1	Also award for 18 : 30 : 42
					Total 3 marks

Q3.

Question	Working	Answer	Mark	Notes	
(a)	7/32 x 100 oe			M1	
		21.9	2	A1	(21.875) accept awrt to 21.9
(b)	4/100 x 32 (=1.28) or 4/100 x 32000000 (=1280000) 32 + "1.28" or 32000000 + "1280000")			M1	M2 for 32 x 1.04 oe or 32000000 x 1.04 oe
				M1	(dep)
				A1	(33.28) accept 33.3, 33000000, 33300000,
		33	3		33280000
					Total 5 marks

Q4.

Q	Working	Answer	Mark	Notes
(a)	$5\sqrt{2} + 8\sqrt{2}$ $= 13\sqrt{2}$			
(b)	$20 - 4\sqrt{3} + 5\sqrt{12} - \sqrt{3}\sqrt{12}$ or $20 - 4\sqrt{3} + 5\sqrt{12} - \sqrt{36}$ or $20 - 4\sqrt{3} + 5\sqrt{12} - 6$		3	M1 for at least 3 correct terms with correct signs or all 4 terms correct without signs
	$\sqrt{12} = \sqrt{4 \times 3}$ or $\sqrt{12} = 2\sqrt{3}$ or $5\sqrt{12} = 5\sqrt{4 \times 3}$ or $5\sqrt{12} = 10\sqrt{3}$			M1 NB. This may be seen before the expansion of the brackets
		show		A1 dep on both method marks for deriving the given answer
				Total 5 marks

Q6.

Question	Working	Answer	Mark	Notes
	$y^2 = ay^2 + n$		5	M1
	$y^2 - ay^2 = n$ or $1 = a + \dfrac{n}{y^2}$ or $1 - a = \dfrac{n}{y^2}$			M1 isolate terms in y^2 or divide through by y^2
	$y^2(1-a) = n$			M1 take out y^2 as a common factor
	$y^2 = \dfrac{n}{1-a}$			M1 y^2 as subject
		$\sqrt{\dfrac{n}{1-a}}$		A1 accept $\sqrt{\dfrac{-n}{a-1}}$
				Total 5 marks

Q8.

Q	Working	Answer	Mark	Notes
a			2	M1 $(2x\pm1)(2x\pm1)$
		$(2x-1)(2x+1)$		A1 cao
b	$\dfrac{4(2x-1)}{(2x-1)(2x+1)} + \dfrac{1}{4x^2-1}[=3]$ or		4	M1 multiply all terms by $(4x^2-1)$ or
	$\dfrac{4(4x^2-1)}{(2x+1)(4x^2-1)} + \dfrac{(2x+1)}{(2x+1)(4x^2-1)}[=3]$ or			correct equation with fractions with a common denominator
	$\dfrac{4(4x^2-1)}{2x+1} + \dfrac{1(4x^2-1)}{4x^2-1} = 3(4x^2-1)$ or			NB $(4x^2-1)$ may be factorised throughout
	$4(4x^2-1) + (2x+1) = 3(2x+1)(4x^2-1)$			
	$4(2x-1)+1 = 3(4x^2-1)$ oe or			M1 correct equation with no fractions
	$4(4x^2-1) + 2x + 1 = 3(2x+1)(4x^2-1)$ oe			
	$12x^2 - 8x = 0$ or $8x - 12x^2 = 0$ or			M1 correct simplified equation with all terms on one side
	$24x^3 - 4x^2 - 8x = 0$ or $8x + 4x^2 - 24x^3 = 0$			
		$0, \dfrac{2}{3}$		A1 dep on M2
				Total 6 marks

Q5.

Question	Working	Answer	Mark	Notes		
(a)		$7(2x-5)$	1	B1		
(b)	$6c-15-2c+8$		2	M1	Any three terms correct.	
		$4c-7$		A1		
(c)		$16e^6$	2	B2	B1 for 16 or e^6 as part of a product	
					or	
					B1 for $4^2 \times e^{2 \times 3}$	
(d)	$2a^2+10a-a-5$		2	M1	Any three terms correct or $2a^2+9a+...$ or $...+9a-5$	
		$2a^2+9a-5$		A1		
						Total 7 marks

Q9.

Q	Working	Answer	Mark	Notes
(a)	2 x 3 x x x = (x + 10)(3x + 20) or $6x^2 = (x + 10)(3x + 20)$ $6x^2 = 3x^2 +50x + 200$		3	M2 If not M2 then M1 for 2 x 3x x x or 2 x $3x^2$ or $6x^2$ or (x + 10)(3x + 20) A1 Dependent on at least M1
(b)	$(3x +10)(x - 20)$ (=0) Marks can be awarded in b) if seen in a) 20 x 3 x 20	x =20 1200	5	M2 or $\quad x = \frac{50 \pm \sqrt{2500+2400}}{6}$ If not M2 then M1 for $(3x \pm10)(x \pm20)$ or $x = \frac{--50 \pm \sqrt{-50^2-4x3x-200}}{2 \times 3}$ condone 1 sign error A1 dep on M1 in b). Ignore negative root $(-3.3$ rec) M1 A1 dep on 1st M1 in b)
				Total 8 marks

Q10.

		3	M1	cos must be selected for use in trig ratio **NOT** Cosine Rule	**or M2** for sin and $\dfrac{\sqrt{"21.36"}}{9.5}$ following correct Pythagoras **or M2** for tan and $\dfrac{\sqrt{"21.36"}}{8.3}$ following correct Pythagoras
use of cos					
$\cos("x") = \dfrac{8.3}{9.5}\ (=0.87...)$ **or** $("x" =)\cos^{-1}\left(\dfrac{8.3}{9.5}\right)$			M1		**or** correct Pythag and then correct use of sine or cosine rule with "21.36"
	29.1		A1	for ans rounding to 29.1 (29.1103...)	
					Total 3 marks

Q11.

The correct answer, unless clearly obtained by an incorrect method, should be taken to imply a correct method.

Question	Working	Answer	Marks	Comments
(a)	Probabilities on branches correct.	$\dfrac{6}{10}, \dfrac{4}{10}, \dfrac{5}{9},$ $\dfrac{4}{9}, \dfrac{6}{9}, \dfrac{3}{9}$	3	B1 for $\dfrac{6}{10}, \dfrac{4}{10}$ oe on LH branches B1 for $\dfrac{5}{9}, \dfrac{4}{9}$ oe on top RH branches B1 for $\dfrac{6}{9}, \dfrac{3}{9}$ oe on bottom RH branches Decimals given on the 2nd set of branches to be to at least 2dp (truncated or rounded).
(b)	$\dfrac{6}{10} \times \dfrac{4}{9}$ or $\dfrac{4}{10} \times \dfrac{6}{9}$ **or** ft from their tree diagram $\dfrac{6}{10} \times \dfrac{4}{9} + \dfrac{4}{10} \times \dfrac{6}{9}$ **or** ft from their tree diagram	$\dfrac{48}{90}$ oe	3	M1 or $\dfrac{6}{10} \times \dfrac{5}{9} + \dfrac{4}{10} \times \dfrac{3}{9} \left(= \dfrac{42}{90} \right)$ M1dep or $1 - "\dfrac{42}{90}"$ A1 Allow 0.53(33...) Note: If all 4 probability products are seen at the ends of the branches on the tree diagram or in lists in the working space for (b), marks can only be awarded in (b) if it is clear which product(s) they are intending to use.
				Total 6 marks

Q12.

The correct answer, unless clearly obtained by an incorrect method, should be taken to imply a correct method.

Question	Working	Answer	Marks	Comments	
(a)	$(25 + 1) \div 2$ or 13 or 12.5		2	M1	or listing scores and clear attempt to find middle value
		2		A1	
(b)	$1 \times 9 + 2 \times 6 + 3 \times 3 + 4 \times 2 + 5 \times 1 + 4 \times 6$ oe (=67)		3	M1	sight of at least 4 products and intention to add
	"67"÷25 or $\dfrac{9+12+9+8+5+24}{25}$ oe (allow one error in a product)			M1dep	for division of sum of products by 25 (can be their 25 if evidence of adding frequencies)
		2.68 or $2\dfrac{17}{25}$		A1	accept 2.7 or 3 if preceded by $\dfrac{67}{25}$
					Total 5 marks

Q13.

Q	Working	Answer	Mark	Notes	
		1, 8, 9	2	B2	B1 for 2, 8, 8 or 0, 8, 10 or for three numbers with a mean of 6 or a median of 8 or 6×3 (=18)
					Total 2 marks